ASYMPTOTICAL METHOD TO SOLUTION THE IDENTIFICATION PROBLEM FOR DETERMINING THE PARAMETERS OF DISCRETE DYNAMICAL SYSTEMS

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1. Introduction

In the paper the dynamical system in the discrete case [1,4] is considered. In this process the motion of an object is described by a system of nonlinear difference equations, the right side of which, in addition to phase coordinates, includes an unknown constant parameter vector and a small number. Using the quasilinearization method [3], the initial problem is reduced to the system of linear difference equations [2]. Then, on the basis of statistical data for the initial and final conditions, the corresponding quadratic functional is constructed and the functional gradient is derived. A computational algorithm for solving the considered problem is proposed.

2. Main problem

Let the system of discrete dynamical nonlinear equations has the following form

$$y\left(i+1\right)=f\left(y\left(i\right),\alpha,\varepsilon\right),\quad i=\overline{0,N-1} \tag{1}$$

with initial condition

$$y_j\left(0\right)=y_{0j},j=\overline{1,M}, \tag{2}$$

where α - m-dimensional unknown constant vector, ε- small parameter, M,N- given natual numbers, f - n –dimensional function, differentiable with respect to y, α, ε.

It is required to find such vector-parameter $\alpha{=}\tilde{\alpha}$, which the solution of the Cauchy problem (1)-(2) satisfies the given condition

$$y_j\left(N\right)=y_{Nj}\,,\ j=\overline{1,N}. \tag{3}$$

The solution of the problem (1)-(3) can be solved with the method of quasilinearization [2,3]. In the first step we linearize the equation (1). Then selecting some nominal trajectory $y^0(i)$ and the parameter α^0, we assume that (k-1)-th iteration has been already fulfilled. If we linearize the equation (1) with respect to $y^{k-1}(i)$, $\alpha^{k-1}$and ε

$$y^k\left(i+1\right)=\Big(A_0\left(y^{k-1}\left(i\right),\alpha^{k-1}\right)+\varepsilon A_1\left(y^{k-1}\left(i\right),\alpha^{k-1}\right)y^k\left(i\right)+\Big(B_0\left(y^{k-1}\left(i\right),\alpha^{k-1}\right)+$$

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$$+\varepsilon B_1\left(y^{k-1}(i),\alpha^{k-1}\right)\right)\alpha^k+\left(C_0\left(y^{k-1}(i),\alpha^{k-1}\right)+\varepsilon C_1\left(y^{k-1}(i),\alpha^{k-1}\right)\right), \quad (4)$$

where

$$A_0\left(y^{k-1}(i),\,\alpha^{k-1}\right)=\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial y(i)},\;\; A_1\left(y^{k-1}(i),\,\alpha^{k-1}\right)=\frac{\partial^2 f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial y\partial\varepsilon},$$

$$B_0\left(y^{k-1}(i),\,\alpha^{k-1}\right)=\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial\alpha},\;\; [B_1\left(y^{k-1}(i),\,\alpha^{k-1}\right)=\frac{\partial^2 f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial y\partial\varepsilon},$$

$$C_0=f\left(y^{k-1}(i),\alpha^{k-1},0\right)-\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial y(i)}y^{k-1}(i)-\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial\alpha}\alpha^{k-1},$$

$$C_1=\frac{\partial f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial\varepsilon}-\frac{\partial^2 f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial y\partial\varepsilon}y_{k-1}(i)-\frac{\partial^2 f\left(y^{k-1}(i),\alpha^{k-1},0\right)}{\partial\alpha\partial\varepsilon}\alpha^{k-1}.$$

$y^k(N)$ from the equation (4) has the form

$$y^k(N)=\left(\Phi_0^{0k-1}(i)+\varepsilon\Phi_0^{1k-1}(i)\right)y^k(0)+\left(\Phi_1^{0k-1}(i)+\varepsilon\Phi_1^{1k-1}(i)\right)\alpha^k+$$
$$+\left(\Phi_2^{0k-1}(i)+\varepsilon\Phi_2^{1k-1}(i)\right) \quad (5)$$

and

$$\Phi_0^{0k-1}(i)=\prod_{i=N-1}^{0}A_0^{k-1}(i),\Phi_0^{1k-1}(i)=A_1^{k-1}(N-1)\prod_{i=N-2}^{0}A_0^{k-1}(i),$$

$$\Phi_1^{0k-1}(i)=\sum_{P=1}^{N-1}\left(\prod_{i=N-1}^{P}A_0^{k-1}(i)\right)B_0^{k-1}(P-1)+B_0^{k-1}(N-1),$$

$$\Phi_1^{1k-1}(i)=\sum_{P=1}^{N-1}\left(\prod_{i=N-1}^{P}A_0^{k-1}(i)\right)B_1^{k-1}(P-1)+\sum_{P=1}^{N-1}\left(A_1^{k-1}(N-1)\left(\prod_{i=N-2}^{P}A_0^{k-1}(i)\right)\right)\times$$
$$\times B_0^{k-1}(P-1)+B_1^{k-1}(N-1),$$

$$\Phi_2^{0k-1}(i)=\sum_{P=1}^{N-2}\left(\prod_{i=N-1}^{P}A_0^{k-1}(i)\right)C_0^{k-1}(P-1)+C_0^{k-1}(N-1),$$

$$\Phi_2^{1k-1}(i)=\sum_{P=1}^{N-2}\left(\prod_{i=N-1}^{P}A_0^{k-1}(i)\right)C_1^{k-1}(P-1)+A_1^{k-1}(N-1)\left(\prod_{i=N-2}^{P}A_0^{k-1}(i)\right)C_0^{k-1}(P-1)+$$
$$+C_1^{k-1}(P-1).$$

Then we construct the following quadratic functional in the iteration –th

$$I^k=\sum_{s=1}^{n}\left(y_s^k(N)-y_{NS}^k\right)^T A\left(y_S^k(N)-y_{NS}^k\right), \quad (6)$$

where the symbol T means the operation of transpose, A is a $n\times n$ dimensional constant symmetric weight matrix, $y_s^k(N)$ is a $n\times 1$ dimensional vector of observation defined by (5),

y^k_{NS} -$n \times 1$ dimensional vector. Then the solution of the stated problem is reduced to the problem: Find a constant vector α, by which the solution of the equation (1) with initial data (2) minimizes the functional (6).

After substituting $y^k(N)$ from (5) into (6) the gradient $\frac{\partial J^k}{\partial \alpha}$ has the form

$$\frac{\partial y^k}{\partial \alpha} = \sum_{s=1}^{n} \Big[{\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{0S}}^{k-1}(i)\, y^k_s(0) - {\Phi^0_{1S}}^{k-1}(i)\, A y^k_{NS} + {\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{2S}}^{k-1}(i) +$$

$$+\varepsilon \Big({\Phi^1_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{0S}}^{k-1}(i)\, y^k_s(0) + {\Phi^0_{1S}}^{k-1\prime}(i)\, \mathrm{A} {\Phi^1_{0S}}^{k-1}(i)\, y^k_s(0) + {\Phi^1_{1S}}^{k-1\prime}(i)\, \mathrm{A} {\Phi^0_{2S}}^{k-1}(i) +$$

$$+{\Phi^1_{2S}}^{k-1\prime}(i)\, A {\Phi^0_{1S}}^{k-1}(i) \Big) + \Big({\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{1S}}^{k-1}(i) + 2\varepsilon {\Phi^0_{1S}}^{k-1\prime}(i)$$
$$\mathrm{A}\Phi\ {}^{1}_{1S}{}^{k-1}(i) \Big)\, \alpha^k_s \Big]. \tag{7}$$

Finally, seekingα^k in the form $\alpha^k \approx \alpha^k_0 + \varepsilon\ \alpha^k_1$ and equating the expression (7) to zero we define $\alpha^k_0,\ \alpha^k_1$ in the following form

$$\alpha^k_0 = -\sum_{s=1}^{n} \Big\{ \Big({\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{1S}}^{k-1}(i) \Big)^{-1} \Big({\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{0S}}^{k-1}(i)\, y^k_s(0) -$$
$$-{\Phi^0_{1S}}^{k-1}(i)\, \mathrm{A}\ y^k_{NS} + {\Phi^0_{1S}}^{k-1\prime}(i)\, \mathrm{A} {\Phi^0_{2S}}^{k-1}(i) \Big\}, \tag{8}$$

$$\alpha^k_1 = -\sum_{s=1}^{n} \Big\{ \Big({\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{1S}}^{k-1}(i)\, (2\varepsilon + 1) \Big)^{-1} \Big({\Phi^1_{1S}}^{k-1\prime}(i)\, A\ {\Phi^0_{0S}}^{k-1}(i)\, y^k_s(0) +$$

$$+{\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^1_{0S}}^{k-1}(i)\, y^k_s(0) + {\Phi^1_{1S}}^{k-1\prime}(i)\, \mathrm{A} {\Phi^0_{2S}}^{k-1}(i) + {\Phi^1_{2S}}^{k-1\prime}(i)\, \mathrm{A} {\Phi^0_{1S}}^{k-1}(i) -$$

$$-{\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^1_{1S}}^{k-1}(i) \Big({\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{1S}}^{k-1}(i) \Big)^{-1} \Big({\Phi^0_{1S}}^{k-1\prime}(i)\, A {\Phi^0_{0S}}^{k-1}(i)\ y^k_s(0) -$$

$$-{\Phi^0_{1S}}^{k-1}(i)\, Ay\ {}^{k}_{NS} + \Phi^0_{1S}\ {}^{k-1\prime}(i)\, A {\Phi^0_{2S}}^{k-1}(i) \Big) \Big\}. \tag{9}$$

Keywords: Nonlinear Discrete Equation, the Method of Quasilinearization, the Gradient of the Functional, Identification.

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References

[1] Aliev F.A., Hajieva N.S., Namazov A.A., Safarova N.A., Identification problem for determining the parameters of a discrete dynamic system, *International Applied Mechanics,* Vol.55, No.1, 2019, pp.110-116.

[2] Ashyralyev A., Agirseven D., Agarwal R.P., Stability estimates for delay parabolic differential and difference equations, *Appl. Comput. Math.,* Vol.19, No.2, 2020, pp.175-204.

[3] Bellman P.E., Kalaba P.E., *Quasilinearization and Nonlinear Boundary Problems,* M., Mir, 1968, 153 p. (in Russian).

[4] Magagula V.M., Motsa S.S., Sibanda P., A new bivariate spectral collocation method with quadratic convergence for systems of nonlinear coupled differential equations, *Appl. Comput. Math.,* Vol.18, No.2, 2019, pp.113-122.